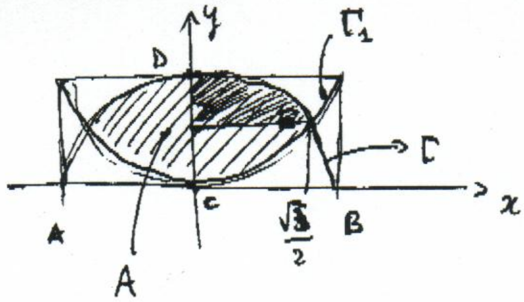
PROBLEMA 2.

1) Scegliamo un riferimento cartesiano come segue:



$C = (0,0)$

$A = (-1,0)$

$B = (1,0)$

$D = (0,1)$

$E = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Per calcolare A trovo anzitutto E.

$$\begin{cases} x^2+y^2=1 \\ x^2+y^2-2y=0 \end{cases} \equiv \begin{cases} \Gamma \longrightarrow \text{centro in } C \text{ e raggio } 1. \\ \Gamma_1 \longrightarrow \text{centro in } D \text{ e raggio } 1. \end{cases}$$

$\Downarrow$

$$1-2y=0 \Rightarrow y=\frac{1}{2} \;;\; x=\pm\sqrt{1-\frac{1}{4}} \Rightarrow E=\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right).$$

Per simmetria si ha $A = 4\left[\int_0^{\sqrt{3}/2} \sqrt{1-x^2}\,dx - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right] =$

$$4\left[\int_0^{\frac{\pi}{3}} \cos^2 t\,dt - \frac{\sqrt{3}}{4}\right] = 4\left[\frac{\sqrt{3}}{8}+\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right] = \frac{-3\sqrt{3}+4\pi}{6}.$$

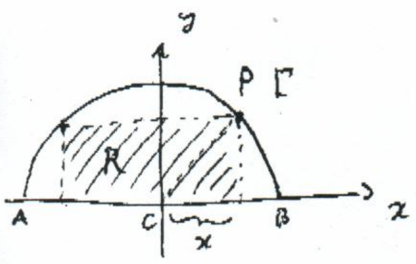
$x = \operatorname{sen} t$

per parti

$$\int_0^{\frac{\pi}{3}} \cos^2 t\,dt = \operatorname{sen} t \cos t \Big|_0^{\pi/3} + \int_0^{\frac{\pi}{3}} 1-\cos^2 t\,dt =$$

$$= \frac{\sqrt{3}}{4}+\frac{\pi}{3} - \int_0^{\pi/3} \cos^2 t\,dt.$$

2)



Per $x \in [0,1]$ si ha

$$A_R = 2x\sqrt{1-x^2}.$$

$$A_R(0)=0 \;;\; A_R(1)=0.$$

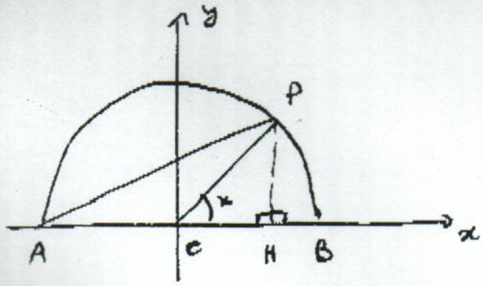
A_R è derivabile e si ha $A'_R(x) = \dfrac{2-4x^2}{\sqrt{1-x^2}}$. $\quad A'_R = 0 \iff x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\sqrt{2}}{2}$ accettabile.

$A'_R < 0$ con $x > \frac{\sqrt{2}}{2}$, $A'_R > 0$ con $x < \frac{\sqrt{2}}{2}$ $\Rightarrow$ $x = \frac{\sqrt{2}}{2}$ è punto di massimo.

Il rettangolo di area massima è costituito da due quadrati; l'altezza è dunque metà della base.

b) . Caso $x \in [0, \frac{\pi}{2})$



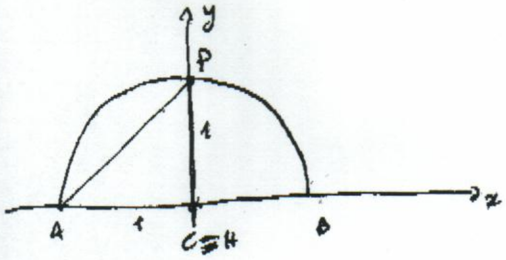
$PH = \operatorname{sen} x$
$CH = \cos x$

$$S_1 = A_{APH} = (1+\cos x)\frac{\operatorname{sen} x}{2}$$

$$S_2 = A_{PCH} = \frac{\cos x \cdot \operatorname{sen} x}{2}$$

$$\Rightarrow f(x) = \frac{1+\cos x}{\cos x}$$ (ha senso anche se $\operatorname{sen} x = 0$).

• Caso $x = \frac{\pi}{2}$

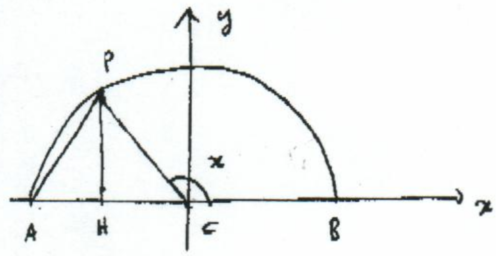


$S_1 = A_{APH} = \frac{1}{2}$

$S_2 = A_{PCH} = 0$

$\frac{S_1}{S_2}$ non è definito.

. Caso $x \in (\frac{\pi}{2}, \pi]$



$PH = \operatorname{sen}(\pi - x) = \operatorname{sen} x$

$CH = \cos(\pi - x) = -\cos x$

$$S_1 = A_{APH} = (1+\cos x) \cdot \frac{\operatorname{sen} x}{2}$$

$$S_2 = A_{PCH} = -\frac{\cos x \cdot \operatorname{sen} x}{2}$$

$$\Rightarrow \frac{S_1}{S_2} = f(x) = -\frac{1+\cos x}{\cos x}.$$

c) Studio di $f(x) = \begin{cases} \frac{1+\cos x}{\cos x} & x \in [0, \frac{\pi}{2}) \\ -\frac{1+\cos x}{\cos x} & x \in (\frac{\pi}{2}, \pi] \end{cases} = \frac{1+\cos x}{|\cos x|}.$

Comincio a studiare $g(x) = \frac{1+\cos x}{\cos x}$ su $\mathbb{R}$; g è periodica di periodo 2π. La studio solo su $[0, 2\pi]$.

D: $x \neq \frac{\pi}{2}$, $x \neq \frac{3}{2}\pi$.

$g > 0$ con $x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{3}{2}\pi, 2\pi\right]$.

$g = 0$ con $x = \pi$.

$\lim_{x \to \frac{\pi}{2}^-} g = +\infty$; $\lim_{x \to \frac{\pi}{2}^+} g = -\infty$; $\lim_{x \to \frac{3}{2}\pi^-} g = -\infty$; $\lim_{x \to \frac{3}{2}\pi^+} g = +\infty$.

$x = \frac{\pi}{2}$, $x = \frac{3}{2}\pi$ asintoti verticali.

$g'(x) = \frac{\operatorname{sen} x}{\cos^2 x}$.

$g' > 0$ con $x \in [0, \pi) \setminus \left\{\frac{\pi}{2}\right\}$. g strett. crescente.

$g' < 0$ con $x \in (\pi, 2\pi] \setminus \left\{\frac{3}{2}\pi\right\}$. g strett. decrescente.

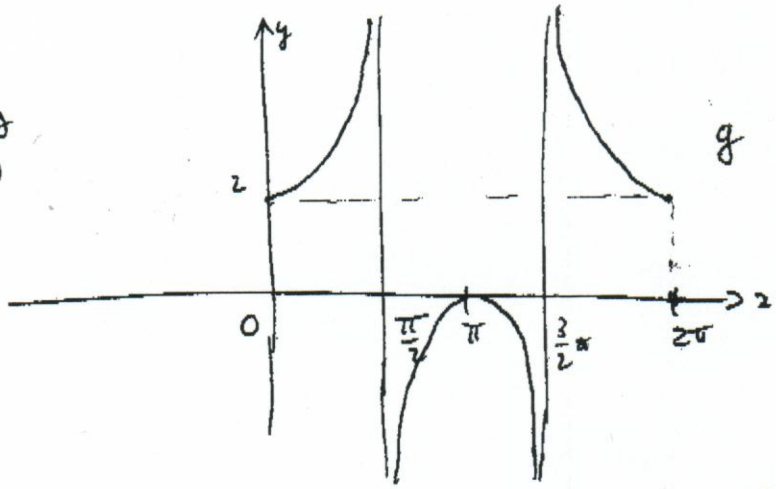
$g' = 0 \iff x = \pi$. pto di max locale.

$g'_+(0) = g'_-(2\pi) = 0$.

$g''(x) = \frac{\cos^2 x + 2 \operatorname{sen}^2 x}{\cos^3 x}$

. g è convessa dove $g > 0$; g è concava dove $g < 0$. g non ha flessi.

. Grafico di g in $[0, 2\pi)$



. Grafico di $f(x) = \begin{cases} g(x) & x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{3}{2}\pi, 2\pi\right] \\ -g(x) & x \in \left(\frac{\pi}{2}, \frac{3}{2}\pi\right). \end{cases}$ in $[0, 2\pi)$