

PROBLEMA 5.

$$P(x) = ax^3 + bx^2 + cx + d$$

$$P(0) = 0 \Leftrightarrow d = 0$$

$$P'(x) = 3ax^2 + 2bx + c$$

$$P'(0) = 0 \Leftrightarrow c = 0.$$

$$\left. \begin{array}{l} P(0) = 0 \Leftrightarrow d = 0 \\ P'(0) = 0 \Leftrightarrow c = 0 \end{array} \right\} \rightarrow P(x) = ax^3 + bx^2$$

$$P(1) = 0 \Leftrightarrow a = -b \Rightarrow P(x) = ax^3 - ax^2$$

$$\int_0^1 P(x) dx = \frac{1}{12} \Leftrightarrow a \int_0^1 (x^3 - x^2) dx = \frac{1}{12}$$

$$\int_0^1 (x^3 - x^2) dx = \left. \frac{x^4}{4} - \frac{x^3}{3} \right|_0^1 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = -\frac{1}{12}$$

$$\Rightarrow \int_0^1 P(x) dx = \frac{1}{12} \Leftrightarrow a = -1$$

Por lo tanto $P(x) = x^2 - x^3$.