Calcolare le derivate parziali prime delle funzioni

(1) $f(x,y) = y - \log(x+y^2)$;

(2) $f(x,y,z) = x^y + z^2$;

(3) $f(x,y,z) = (xy)^z$;

(4) $f(x,y) = \sqrt{x^2+y^2}$;

(5) $f(x,y) = \sin(y^4\cos(3x))$;

(6) $f(x,y) = e^{-x^2\arctan(2y^3)}$;

(7) $f(x,y,z) = x^3\cos(y^2z^4)$;

(8) $f(x,y,z) = z^3\log(1+x^6y^4)$;

(9) $f(x,y,z,t) = t^4\cos x + y^3e^{-z}$.

SOLUZIONE.

(1)

$$\frac{\partial f}{\partial x}(x,y) = -\frac{1}{x+y^2} \quad \frac{\partial f}{\partial y}(x,y) = 1 - \frac{2y}{x+y^2}.$$

(2)

$$\frac{\partial f}{\partial x}(x,y,z) = yx^{y-1} \quad \frac{\partial f}{\partial y}(x,y,z) = x^y\log x \quad \frac{\partial f}{\partial z}(x,y,z) = 2z.$$

(3)

$$\frac{\partial f}{\partial x}(x,y,z) = y^z z x^{z-1} \quad \frac{\partial f}{\partial y}(x,y,z) = x^z z y^{z-1} \quad \frac{\partial f}{\partial z}(x,y,z) = (xy)^z\log(xy).$$

(4)

$$(x,y)\neq(0,0) \Longrightarrow \frac{\partial f}{\partial x}(x,y) = \frac{x}{\sqrt{x^2+y^2}} \quad \frac{\partial f}{\partial y}(x,y) = \frac{y}{\sqrt{x^2+y^2}}.$$

Se $(x,y)=(0,0)$, allora calcoliamo direttamente

$$\lim_{t\to 0}\frac{f((0,0)+t(1,0))-f(0,0)}{t} = \lim_{t\to 0}\frac{|t|}{t},$$

che non esiste. Dunque f non è derivabile in $(0,0)$.

(5)

$$\frac{\partial f}{\partial x}(x,y) = -3\cos(y^4\cos(3x))y^4\sin(3x) \quad \frac{\partial f}{\partial y}(x,y) = \cos(y^4\cos(3x))4y^3\cos(3x).$$

(6)

$$\frac{\partial f}{\partial x}(x,y) = e^{-x^2\arctan(2y^3)}(-2x)\arctan(2y^3) \quad \frac{\partial f}{\partial y}(x,y) = e^{-x^2\arctan(2y^3)}\frac{-x^2}{1+4y^6}6y^2.$$

(7)

$$\frac{\partial f}{\partial x}(x,y,z) = 3x^2\cos(y^2z^4) \quad \frac{\partial f}{\partial y}(x,y,z) = -2yx^3z^4\sin(y^2z^4)$$

$$\frac{\partial f}{\partial z}(x,y,z) = -4x^3y^2z^3\sin(y^2z^4).$$

(8)

$$\frac{\partial f}{\partial x}(x,y,z) = z^3 \frac{1}{1+x^6y^4} 6x^5y^4 \quad \frac{\partial f}{\partial y}(x,y,z) = z^3 \frac{1}{1+x^6y^4} 4x^6y^3$$
$$\frac{\partial f}{\partial z}(x,y,z) = 3z^2 \log(1+x^6y^4).$$

(9)

$$\frac{\partial f}{\partial x}(x,y,z,t) = -t^4 \sin x \quad \frac{\partial f}{\partial y}(x,y,z,t) = 3y^2 e^{-z}$$
$$\frac{\partial f}{\partial z}(x,y,z,t) = -y^3 e^{-z} \quad \frac{\partial f}{\partial t}(x,y,z,t) = 2t \cos x.$$