198. An Introduction to Game Theory and its Applications

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I Introduction

Game theory is formally defined as "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers" (Myerson, 2001, p. 1). One alternative definition, proposed "as a more descriptive name for the discipline", is "interactive decision theory" (Aumann, 2008, Abstract). In other words, it is the analysis (by means of mathematical reasoning) of a conflict of interest to find the optimal choices for reaching the desired outcome, under given conditions. Basically, it is the study of the ways to 'win' in a situation given certain circumstances.
Putting its limitations to one side, game theory has been profitably applied to many situations in the field of economics, biology, sociology, and political sciences, to predict important trends. This paper aims to offer a brief, clear overview of the main aspects of game theory and its wider applications.

II Core concepts

i. Game and players

The object of studying in game theory is the 'game', which is defined as a formal model of an interactive situation in which at least one agent can maximise his utility by anticipating the responses to his actions of one (or more) other agents. A game normally involves several agents (which are referred to as 'players'), but some require only one player (so-called 'decision problems'). The 'formal definition' of a game offers information about the players, their preferences, the information and the strategic actions available to them, and their influence on the outcome.

ii. Rationality

The most significant (and conceivably one of the most controversial) assumption of game theory is that the players are 'rational'. Players are referred to as 'rational' if they have precise and consistent preferences over the set of possible outcomes and are able to faultlessly determine and adopt the best available strategy to reach them.
If taken literally, the assumption of 'rationality' is incontestably an unrealistic one, and – if applied to specific cases – it may produce results seemingly at odds with reality. Game theorists are well-aware of the limitations imposed by this assumption. Indeed, for this reason, there are many research groups studying the implications of a "less demanding form of rationality" (known as "bounded rationality").

iii. Move

A 'move' is defined as the way in which a game progresses between states through exchange of information. The moves available to each player are defined by the rules of the game: they can be the result of a choice or made by chance; they may be made in consecutive fashion, or may occur concurrently for all players, or continuously for a single player until he reaches a certain state or declines to move further. In particular, 'simultaneous games' are games where both players move simultaneously (or, if they do not move simultaneously, each player chooses his action without knowledge of the actions chosen by the other players). On the contrary, a 'sequential game' is a game where one player chooses his action before the others choose theirs (NB: the later players must have some information of the first's choice, otherwise the difference in time would have no strategic effect).

iv. Information

A game is said to have 'perfect information' when at any point in time only one player makes a move, and he is aware of all the actions made until that moment. Only sequential games can be games of perfect information, since players in simultaneous games do not know the actions of the other players. However, most games studied in game theory are imperfect-information games. The concept of 'perfect information' is frequently confused with the similar one of 'complete information': the attribute 'complete information' implies that every player is aware of the strategies and the payoffs available to the other players, but not necessarily of the actions taken by them.

v. Payoff

In any game, the payoffs are numbers which represent the 'motivations' of players: in fact, they may represent profit or other continuous measures (i.e. 'cardinal payoffs'), or may simply rank the desirability of the outcomes (i.e. 'ordinal payoffs').

vi. Strategy

A strategy defines a set of moves or actions a player will follow in a given game.

vii. Dominating strategy

A strategy 'dominates' another strategy if it always provides a better payoff to that player, regardless of the other players' actions. A strategy is said to 'weakly dominate' another one if it is at least as good.

viii. Nash equilibrium

A 'Nash equilibrium', also known as 'strategic equilibrium', is a list of strategies, one for each player, which has the property that no player can unilaterally change his strategy and obtain a better payoff.

ix. n-person games

Games can be classified according to certain significant features. The most straightforward one is the number of players (it must be noticed that a player need not be an individual: it might be a nation, or a team comprising many people with shared interests). A game can be classified as being a one-person, two-person, or n-person (with n greater than two) game.

x. Constant-sum and variable-sum games

'Constant-sum games' are games of 'total conflict' (also known as games of 'pure competition'), in which the sum of all players' payoffs is the same for any outcome. This condition implies that a gain for one participant is always at the expense of another. For instance, poker is a constant-sum game for the combined wealth of the players remains constant, although its distribution can shift during the game.
On the other side, in a 'variable-sum game', the sum of all players' payoffs is not constant (and may vary depending on the strategies adopted by them). Therefore, players in constant-sum games have completely opposed interests, whereas in variable-sum games they may all be winners or losers.

xi. Zero-sum and non-zero-sum games

A zero-sum game, which is a particular type of constant-sum games, is a model of a situation in which a participant's gain (or loss) is exactly balanced by the losses (or gains) of the other participant(s): therefore, if the total gains of the participants are added up, and the total losses are subtracted, they will amount to zero.
On the other side, non-zero-sum games describe a situation in which the interacting parties' summed gains and losses are either less than or more than zero.

xii. Cooperative and non-cooperative games

Variable-sum games can be further categorised as being either 'cooperative' or 'non-cooperative'. In cooperative games players can communicate and, more importantly, make binding agreements; in non-cooperative games players can communicate, but they cannot stipulate such deals.

xiii. Normal (or strategic) and extensive form

A game can be described either in 'normal' form or in 'extensive' form. The strategic (or normal) form is a matrix representation of a simultaneous game. The payoffs are illustrated by a 'payoff matrix',

wherein each row refers the strategy of one player and each column to the strategy of the other player. The matrix entry at the intersection of each row and column gives the outcome of each player choosing the corresponding strategy.
On the other hand, the extensive form (also known as 'game tree') is a graphical representation of a sequential game. It provides information about the players, payoffs, strategies, and the order of moves. The game tree is made up of nodes (or vertices), representing the points at which players can take actions, connected by edges, representing the actions that can be taken at that node. The initial node represents the first decision to be made, and every set of edges from the first node eventually arrives at a terminal node ("an end" of the game). Each terminal node is labelled with the payoffs earned by each player (if the game ends there).

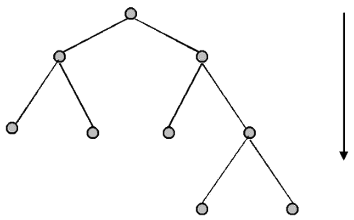

Figure 1

III A simple example

The 'Prisoner's Dilemma' — a non-zero-sum game — is a canonical example of a game analysed in game theory. It was originally shaped by Merrill Flood and Melvin Dresher in 1950, but it was Albert W. Tucker who formalised it.

i. The Prisoner's Dilemma

The name of the Prisoner's Dilemma game derives from the following hypothetical situation classically used to illustrate it. Suppose that the police have arrested two people whom they know have committed a robbery together. However, they lack enough admissible evidence for a conviction, but they do have enough evidence to send each prisoner away for two years for the theft of the getaway car. Having separated both prisoners, the inspector makes the following offer to each of them.: if you testify for the prosecution against the other, and he does not also confess, then you will go free and he will be condemned to ten years. If you both confess, you will each receive a five-year sentence. If neither of you confess, then you will each get two years for the auto theft.
The two players in the game can choose between two moves, either 'cooperate' or 'defect', without having information about which will be the other's 'move'. The key idea is that each player gains if both cooperate, but if only one of them cooperates, the other one, who defects, will gain more. If both defect, they both lose (or at least gain very little) but not as much as the cooperator whose cooperation is not returned.
The situation can be described by a payoff matrix, wherein each cell gives the payoffs to both players for each combination of actions.

Go free ≫ 4
2 years ≫ 3
5 years ≫ 2
10 years ≫ 0

		Prisoner B (Player II)	
		Stay Silent	**Confess**
Prisoner A (Player I)	**Stay Silent**	3,3	0,4
	Confess	4,0	2,2

Table 1

If Player II confesses then Player I obtains a payoff of 2 by confessing and a payoff of 0 by refusing to testify. If Player II refuses to defect, then Player I gets a payoff of 4 by confessing and a payoff of 3 by staying silent. Consequently, in this game, regardless of the other player's choice, each player always receives a better payoff by defecting: in other words, defecting is the 'strictly dominant' strategy. Hence, both players will confess, and both will go to prison for five years.

ii. Solution concepts and equilibria

In the Prisoner's Dilemma, the outcome represented as mutual defection is said to be the 'solution' of the game. Borrowing a term form economists and physicists, game theorists refer to the solutions of games as 'equilibria'. Indeed, when a physical system is said to be in equilibrium, it means that it is in an endogenously stable state: that is, all the forces internal to the system balance each other out, therefore leaving it 'at rest' until and unless it is perturbed by some external forces. Likewise, economists read economic systems as being networks of mutually constraining (often causal) relations – just like physical systems – and the equilibria of such systems are then their endogenously stable states.

What has been referred to as the 'solution' of the Prisoner's Dilemma is the unique Nash equilibrium of the game (where the 'Nash' refers to John Nash, the Nobel Laureate mathematician). Nash equilibrium applies – or fails to apply, as the case might be – to whole sets of strategies: a set of strategies is a Nash equilibrium if no player can improve his payoff, given the strategies of all other players in the game, by changing his strategy.

It is possible to specify one class of games in which Nash equilibrium is always not only necessary but sufficient as a solution concept: these are 'finite' (i.e. with finitely many players, each of which has a finite set of strategies) perfect-information games that are also zero-sum. However, most games do not have this property.

IV Applications of game theory

i. Description and modelling of population dynamics

A well-known use of game theory is to describe and model the human populations' behaviour. In fact, some researchers are positive that by finding the equilibria of certain games they can predict the behaviour of an actual human population, when confronted with situations analogous to the ones in the games studied. However, this particular view of game theory has recently come under criticism, mostly because in the real world the assumptions of game theorists are often violated: human behaviour often deviates from 'rationality' for several reasons (e.g. altruism).

ii. Economics and business

Game theory is an effective method used in mathematical economics and business for modelling the patterns of behaviour of interacting agents. Its applications comprise a wide range of economic phenomena such as auctions, bargaining, fair division, social network formation, voting systems (Tesfatsion, 2006), and can be also found in areas such as experimental economics (V. L. Smith, 1992, pp. 241-282), behavioural economics (Camerer , 1997), and political economy (Shubik, 1981). In these kinds of models, it happens quite often that the payoffs represent money, which most probably corresponds to an individual's utility.

iii. Political science

The application of game theory in political science is focused on the areas of fair division, political economy, war bargaining, and social choice theory. In each of these fields, scholars have developed models in which the players are (for example) voters, states, special interest groups, or politicians (see Downs, 1957).

iv. Biology

John Maynard Smith (a British theoretical evolutionary biologist and geneticist) — in the preface to *Evolution and the Theory of Games* (1982) — writes, "paradoxically, it has turned out that game theory is more readily applied to biology than to the field of economic behaviour for which it was originally designed". Certainly, in biology, game theory has been used to analyse many seemingly incongruous natural phenomena.

One of the main applications of game theory in biology is the study of the so-called 'biological altruism', a behaviour that occurs when an individual ('the donor') performs an action in order to help another organism ('the recipient') with no apparent advantage (or even at a cost) to itself. The costs and benefits are calculated in terms of 'reproductive fitness' (i.e. the expected number of progeny): by behaving self-sacrificingly, an individual reduces the number of offspring it is likely to produce itself, but increases the number of progeny that other animals are likely to produce.

From a Darwinian perspective, the existence of altruism in nature seems perplexing and incongruous: natural selection should lead individuals to behave in order to increase their own chances of survival and reproduction, instead of those of others. Yet instances of altruistic patterns of behaviour can be found in various species ranging from vampire bats that regurgitate blood they have gained and donate it to group members who have failed to find food, to Vervet monkeys that warn group members of a predator's approach, even if it endangers that individual's chances of survival. Moreover, in social insect colonies (e.g. ants, wasps, bees and termites), it happens that sterile workers devote their entire lives to other duties, such as protecting the queen, constructing and defending the nest, looking for food, and nursing the larvae.

Arguably, the problem of altruism is closely associated with questions about the level at which natural selection acts: if selection acts exclusively at the individual level, favouring some individuals over others, then it seems clear that altruism cannot evolve: altruists are at a selective disadvantage compared to the egoistic members of their group since behaving altruistically is detrimental for the individual itself, by definition. However, the fitness of the group as a whole will be enhanced by the presence of altruists, as a group composed of many altruists may have a survival advantage over a group composed predominantly or exclusively of selfish organisms. Therefore, it has been hypothesised that the altruistic behaviour may evolve by 'between-group selection', despite the fact that, within each group, selection favours 'egoistic' individuals. This idea was first proposed by Darwin himself, and later it was appreciated by the founders of modern neodarwinism, although they questioned the importance of this evolutionary mechanism.

Game theory offers another interesting framework for the evolution of 'reciprocal altruism', by modelling biological interactions by means of so called 'Iterated Prisoner's Dilemma'. Indeed, for biological interactions, it is assumed that the same individuals will interact more than once, and if two players play the Prisoner's Dilemma more than once in succession (and they can remember the previous actions of their opponent and change their strategy accordingly) the resulting game is named 'Iterated Prisoner's Dilemma'. The concept of cooperation and altruism — as it is analysed by evolutionary biology — is close to the notion of 'tit for tat' (an English saying meaning 'equivalent retaliation'), which is an effective strategy first introduced by Anatol Rapoport (in the two tournaments held by Robert Axelrod around 1980 in order to find the best strategy for the Prisoner's Dilemma). An agent adopting this strategy will first cooperate, then subsequently replicate the opponent's previous action: if the opponent has been cooperative, the agent will be cooperative; otherwise, the agent will not be cooperative.

A noteworthy explanation for the evolution of altruistic behaviour which does not necessarily depend on game theory is 'inclusive fitness theory', named and developed by British evolutionary biologist William Donald Hamilton. It explains how altruistic patterns of behaviour could evolve without the need for group-level selection: altruistic genes increase in a population by natural selection only if the cost to the altruistic individual is less than the reproductive benefit of the recipient multiplied by the likelihood of the recipient passing on the altruistic gene to its progeny ("Hamilton's Rule"). Inclusive fitness is often associated with 'kin selection', because closely related organisms more likely share the same genes (in this case, the altruistic gene). Nevertheless, altruism genes can be found in non-related individuals: consequently, 'relatedness' is not considered a strict requirement of inclusive fitness.

v. Philosophy

Philosophers have increasingly become interested in game theory since it offers a way of interpreting the thoughts of philosophers such as Immanuel Kant, Thomas Hobbes, Jean-Jacques Rousseau, and many other social and political theorists.

a) Kant's categorical imperative

Immanuel Kant's categorical imperative, which was meant to be the fundamental principle of morality, declares: "Act only according to that maxim whereby you can at the same time will that it should become a universal law without contradiction" (Immanuel Kant, *Grounding for the Metaphysics of Morals*). In terms of game theory, this statement can be paraphrased as follows: "Choose only a strategy which, if you could will it to be chosen by all the players, would yield a better outcome from your point of view than any other". This statement represents a 'moral solution' to the Prisoner's Dilemma. Only a cooperative choice is acceptable, since the choice of defecting, if "made universal", is in contradiction to one's personal interest.

b) Hobbes's and Rousseau's social contract

Through the use of game theory, Thomas Hobbes' argument for absolute monarchy – afterwards made popular by Jean-Jacques Rousseau – can be made clearer. Hobbes argued that, without some form of external constraint on people's pattern of behaviour, anarchy would ensue and cooperation among people would be impossible (as people act only to maximise individual prosperity instead of the welfare of their society). Surely, there will exist altruists who limit their self-interests for the good of others. Nevertheless, if even one self-interested person exists, he will be able to profit from the altruist's constraints: as a result, if there is just one narrowly self-interested person, no altruist will survive (unless he becomes egoistic too). Obviously, in such an environment – known as a 'State of Nature' – a person has to be always ready to pre-empt attacks in order to maximise his own welfare. Each such conflict between people in a State of Nature has been named as 'Hobbesian Dilemma', which, in the field of game theory, has the same structure as a 'Prisoner's Dilemma'. Hobbes believed that the 'Hobbesian Dilemma' results in a State of Nature because morality is an unstable enforcer of social cooperation. However, according to Hobbes, as cooperation among people is biologically necessary, a stable enforcer must exist, and an all-powerful sovereign represents the best form of social enforcement.

V Conclusion

An enormous range of further applications of game theory has been developed, and, regardless of the brevity of the introduction, hopefully, it has been provided enough to persuade the reader of the remarkable, continuously expanding utility of this tool.
The readers whose appetite for more has been aroused should find that they now have a sufficient grasp of the rudiments to be able to work through a large literature on this topic, of which some highlights are listed below.

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